Technical Notes

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Model of the Turbulent Burst Phenomenon: Predictions for Eddy Viscosity

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Introduction

IN the simplest type of surface renewal model of the turbulent burst phenomenona[1-4] the inrush process is envisioned to reach the wall itself, with the unsteady molecular transport within the wall region assumed to be essentially one-dimensional (in space). This elementary type of turbulent burst model is very useful for approximating the dimensionless mean velocity distribution u^+ and the dimensionless mean temperature distribution T^+ for common gases and liquids with low to moderate values of the Prandtl number Pr. But this approach is unacceptable for developing predictions for ϵ_m and ϵ_H very near the wall (and T^+ for fluids with high values of Pr) because of the significance of the unreplenished layer of fluid that resides at the wall surface. To account for the important effects of this layer of fluid on turbulent transport within the wall region, the surface rejuvenation model has been developed.[5-7]

The surface rejuvenation modeling equations for the instantaneous transfer of momentum within fluid near the wall between the inrush and ejection phases for fully turbulent flow with mild pressure gradients is given by[7]

$$\frac{\partial u}{\partial \theta} = \nu \frac{\partial^2 u}{\partial y^2} \tag{1}$$

$u(0,y) = U_{ic} + [g(y) - U_{ic}]U(y-H)$, $\quad u(\theta,0) = 0$, and $u(\theta,y_M) = u_M(\theta)$, where θ is the instantaneous time, U_{ic} the axial velocity of the inrushing fluid, $g(y)$ the instantaneous velocity distribution within the wall region at the instant of inrush, H the instantaneous approach distance, $U(y-H)$ a unit step function, and $u_M(\theta)$ the interfacial condition. The modeling parameter U_{ic} is generally assumed to be uniform.

This system of equations has been put into the mean domain by utilizing a transformation involving the random variables H, g, and θ (Ref. 7). Based on the use of several simplifying modeling approximations, a solution has been obtained for the dimensionless mean velocity distribution u^+ of the form

$$u^+ = \frac{2H^+}{\nu} \frac{J_{2\gamma}(2\gamma) - J_{2\gamma}\{2\gamma \exp[-y^+/(2H^+)]\}}{J_{2\gamma-1}(2\gamma) - J_{2\gamma+1}(2\gamma)} \tag{2}$$

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where $\gamma = \bar{H}\sqrt{\bar{s}/\nu} = H^+\sqrt{\bar{s}\nu}/U^*$. By interfacing this equation with the classical logarithmic law for the turbulent core, γ has been found to be equal to 0.433 and the interface location y_M^+ is 35. With H^+ set equal to 5 on the basis of experimental data by Popovich and Hummel,[8] the dimensionless mean period of the burst phenomenon $U^{*2}/(\bar{s}\nu)$ is equal to 133. This result is compatible with much of the experimental data that have been reported for $\bar{s}$.

Predictions for Reynolds Stress and Eddy Viscosity

Calculations can be made for Reynolds stress $\bar{\tau}_t$ and eddy viscosity ϵ_m within the wall region by utilizing the defining relation

$$\bar{\tau} = \bar{\tau}_y + \bar{\tau}_t = \rho(\nu + \epsilon_m)\frac{\partial \bar{u}}{\partial y} \tag{3}$$

where $\bar{\tau}$ is the total apparent mean shear stress and $\bar{\tau}_y$ is the actual mean viscous shear stress. It follows that

$$\frac{\epsilon_m}{\nu} = \frac{\bar{\tau}_t/\bar{\tau}_0}{\partial u^+/\partial y^+} = \frac{\bar{\tau}/\bar{\tau}_0}{\partial u^+/\partial y^+} - 1 \tag{4}$$

where $\bar{\tau}$ is approximately equal to the mean wall shear stress $\bar{\tau}_0$ in the inner region. Based on Eq. (2), $\partial u^+/\partial y^+$ is given by

$$\frac{\partial u^+}{\partial y^+} = [\exp[-y^+/(2H^+)]]J_{2\gamma+1}\{2\gamma\exp[-y^+/(2H^+)]\} - J_{2\gamma}\{2\gamma\exp[-y^+/(2H^+)]\}]/[J_{2\gamma-1}(2\gamma) - J_{2\gamma+1}(2\gamma)] \tag{5}$$

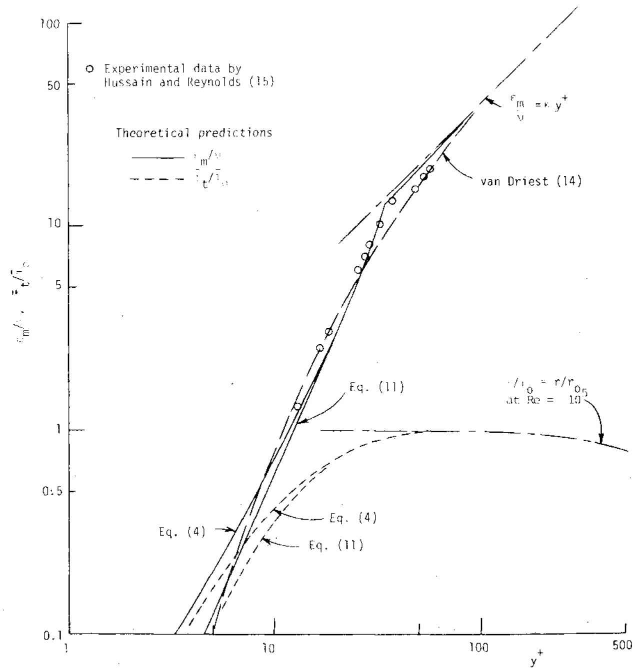

Fig. 1 Predictions for τ_t and ϵ_m.

$\partial u^+/\partial y^+ = 1/(\kappa y^+)$ in the part of the inner region for which $y^+ > y_M^+$. Predictions obtained for $\bar{\tau}_t$ and ϵ_m on the basis of this direct approach are shown in Fig. 1.

The predictions for Reynolds stress are seen to appropriately approach $\bar{\tau}_0$ and $\bar{\tau}$ in the outer part of the wall region well before the effects of pressure gradient (and/or convection) are significant. In the region very near the wall, the predictions for $\bar{\tau}_t$ are approximated by $\bar{\tau}_t/\bar{\tau}_0 = 0.01y^{+1.91}$, which fails to satisfy the minimum limiting criterion established by Elrod[9] and Wasan et al.[10] (i.e., $\bar{\tau}_t/\bar{\tau}_0 \sim y^{+3}$ as $y^+ \to 0$). However, it is in this near wall region in which the present model predictions for $\bar{\tau}_t$ are most sensitive to errors in u^+. For example, a 1% error in $\partial u^+/\partial y^+$ at $y^+ = 1$ will result in $\bar{\tau}_t/\bar{\tau}_0$ error of the order of 100%. Because of modeling approximations involving the statistical distributions in $\bar{H}$ and θ, the inputs for U_{ic} and H^+, and other aspects of the analysis, errors in predictions for $\partial u^+/\partial y^+$ of at least the order of 1% are expected. For that matter, even experimental measurements of the mean gradient in this zone involve errors of this order. It follows that meaningful theoretical predictions or experimental measurements based on the substitution of inputs for $\partial u^+/\partial y^+$ into Eq. (4) can not be made for $\bar{\tau}_t$ in the region very near the wall. These theoretical and experimental limitations pose no practical problem in analyzing momentum transfer. However, the classical extension of theoretical formulations for $\bar{\tau}_t$ in the wall region to heat transfer via the use of assumptions for Pr_t can cause large errors for fluids with high Pr because of the very small thickness of the thermal boundary layer.

Turning to the predictions for ϵ_m, Fig. 1 indicates a basic agreement between the theoretical predictions and the data, with the calculations for ϵ_m approaching the proper limiting equation $\epsilon_m/\nu = \kappa y^+$. As was found in the predictions for $\bar{\tau}_t$, the model predictions for ϵ_m fail to satisfy the limiting equations developed by Elrod and Wasan et al. for the region very close to the wall.

With respect to the questionable predictions for $\bar{\tau}_t$ and ϵ_m very near the wall, the surface rejuvenation model provides the basis for an alternative perspective that eliminates the extreme sensitivity of predictions for these turbulence parameters on small errors in $\partial u^+/\partial y^+$. Preliminary predictions for ϵ_m and ϵ_H obtained on the basis of this new approach and first order modeling approximations are developed in Refs. 11-13. This alternative approach will now be coupled with the above second order analysis for u^+ to obtain more meaningful predictions for $\bar{\tau}_t$ and ϵ_m near the wall.

Alternative Formulation for $\bar{\tau}_t$ and ϵ_m

Following the perspective introduced in Refs. 12 and 13, the mean momentum per unit area carried inwardly across the y-plane by inrushing fluid that moves to within a distance $H(<y)$ of the wall is given by

$$\frac{\mathrm{d}M_H}{\mathrm{d}A} = \int_H^y \rho[U_{ic} - \tilde{u}(1/\bar{s},\xi)]\mathrm{d}\xi \tag{6}$$

where the mean-end-residence-time profile $\tilde{u}(1/\bar{s},\xi)$ can be replaced by the mean velocity distribution $\bar{u}$. To obtain an expression for the mean momentum per unit area which is associated with inrushing fluid that moves to within all distances less than y from the wall, the approach distance distribution $P_H(H)$ is introduced, with the result

$$\frac{\mathrm{d}M}{\mathrm{d}A} = \int_0^y P_H(H)\int_H^y \rho(U_{ic} - \bar{u})\mathrm{d}\xi\mathrm{d}H \tag{7}$$

The mean rate of momentum per unit area carried across the y-plane by the turbulent eddy motion associated with the burst process is given by

$$\frac{\mathrm{d}\dot{M}}{\mathrm{d}A} = \frac{\mathrm{d}M/\mathrm{d}A}{1/\bar{\bar{s}}} \tag{8}$$

where $\bar{\bar{s}}$ is the mean frequency of turbulent exchange below y. $\bar{\bar{s}}$ is expressed in terms of the mean frequency of eddy exchange $\bar{S}$ between the turbulent core and the wall region by[12]

$$\bar{\bar{s}} = \bar{S}\int_0^y P_H(H)\mathrm{d}H \tag{9}$$

By definition, $\mathrm{d}\dot{M}/\mathrm{d}A$ is related to the Reynolds stress $\bar{\tau}_t$ and the eddy diffusivity ϵ_m by

$$\frac{\mathrm{d}\dot{M}}{\mathrm{d}A} = \bar{\tau}_t = \rho\epsilon_m\frac{\partial\bar{u}}{\partial y} \tag{10}$$

It follows that expressions can be written for $\bar{\tau}_t$ and ϵ_m of the form

$$\frac{\epsilon_m}{\nu} = \frac{\bar{\tau}_t/\bar{\tau}_0}{\partial u^+/\partial y^+} = \frac{(\mathrm{d}\dot{M}/\mathrm{d}A)/(\rho U^{*2})}{\partial u^+/\partial y^+} \tag{11}$$

where

$$\frac{\mathrm{d}\dot{M}/\mathrm{d}A}{\rho U^{*2}} = \left[\frac{\bar{S}\nu}{U^{*2}}\int_0^{y^+} P_{H^+}(H^+)\mathrm{d}H^+\right]\left[\int_0^{y^+} P_{H^+}(H^+) \times \int_{H^+}^{y^+}(U_{ic}^+ - u^+)\mathrm{d}\xi^+\mathrm{d}H^+\right] \tag{12}$$

To obtain predictions for $\bar{\tau}_t$ and ϵ_m by means of this modeling approach, Eq. (12) is integrated numerically with u^+ given by Eq. (2) and, based on the physics of the problem, with $\bar{S}$ set equal to $\bar{s}$ (i.e., $U^{*2}/(\bar{S}\nu) = 133$). Predictions obtained for $\bar{\tau}_t$ and ϵ_m by this approach using an exponential distribution in H are shown in Fig. 1. These two approaches produce essentially equivalent predictions for $\bar{\tau}_t$ and ϵ_m in the outer part of the wall region where Eq. (4) is least sensitive to computational errors.

Discussion and Conclusion

Focusing attention on the region very near the wall, the predictions developed for ϵ_m on the basis of the alternative surface rejuvenation formulation are seen in Fig. 1 to lie well below the direct calculations obtained from Eq. (4). In the limit as y^+ becomes small, Eqs. (11) and (12) give $\bar{\tau}_t/\bar{\tau}_0 = \epsilon_m/\nu = 0.00382\ y^{+3}$, which is consistent with the limiting analyses by Elrod[9] and Wasan et al.[10] As indicated earlier, the incompatibility of Eq. (4) and Eqs. (11) and (12) in the region very close to the wall is caused by the extreme sensitivity of Eq. (4) to even slight errors in the modeling predictions for $\partial u^+/\partial y^+$. The alternative perspective upon which Eqs. (11) and (12) are based eliminates this problem. The fact that the two modeling viewpoints give rise to predictions for $\bar{\tau}_t$ and ϵ_m that are compatible in the outer wall region gives evidence of the internal consistency in the general surface renewal modeling approach.

As indicated in the analysis, the mean frequency $\bar{S}$ of fluid exchange between the turbulent core and the wall region has been assumed to be equal to the mean frequency $\bar{s}$ of the turbulent burst process; i.e., $U^{*2}/(\bar{S}\nu) = 133$. This prediction for $\bar{S}$ (or $\bar{s}$) is consistent with much of the experimental data obtained by flow visualization and anemometry techniques for the mean periodicity in turbulent fluctuations in wall turbulence.

The surface rejuvenation model, which accounts for the effect of the unreplenished layer of fluid on the transport of momentum associated with the turbulent burst phenomenon, is shown in the present Note to provide a basis for the prediction of Reynolds stress $\bar{\tau}_t$ and eddy diffusivity ϵ_m within the wall region. The potential usefulness of this general approach is underscored by: 1) the basic consistency between the predictions for ϵ_m and experimental data within the wall region, 2) the appropriateness of the limiting predictions for $\bar{\tau}_t$ and ϵ_m as y^+ becomes large and small, 3) the consistency of the direct and alternative formulation approaches, and 4) compatibility of the predictions for $\bar{s}$ (or $\bar{S}$) with experimental measurements within the wall region. Furthermore, this general approach to modeling wall turbulence can be applied to heat and mass transfer and can be adapted to problems involving transitional turbulence, acceleration, variable properties, natural convection, and other factors.

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Empirical Estimates of Gust Loads on Finite Wings

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I. Introduction

ESTIMATES of the aerodynamic loads on finite aircraft wings in atmospheric gusts are a necessary part of the design exercise for modern aircraft. A variety of theoretical computations based on vortex and/or doublet lattice panel methods[1,2] are the most common means of arriving at these estimates, although experimental data from wind tunnel testing also contribute in the assessment of specific designs.

The availability of wind tunnel data[3,4] for unsteady lift and pitching moment on a variety of finite wing planforms in oscillatory gusts presented the opportunity of devising an empirical relationship which describes the variation of oscillating aerodynamic forces as a function of frequency, freestream velocity, wing sweep angle, and aspect ratio over a range of from 0 to 70 deg in sweep angle and from 1 to 8 in aspect ratio. This Note presents the empirical relationship and its derivation from the raw data.

A gust tunnel of novel design[5] has been used to measure the oscillating lift and pitching moments on a variety of wing planforms subjected to incident oscillatory vertical gusts of varying frequency, amplitude, and freestream velocity. Table 1 displays salient details of the eight wing planforms that were tested. They range from a rectangular wing (A) of aspect ratio 6 to wing F with aspect ratio 4 and a sweep angle of 45 deg. Two of the planforms (G and H) are delta wings with aspect ratios of 2 and 1, respectively. All the wings had straight trailing and leading edges and no center body at the wing root. The nondelta planforms were built with symmetric NACA 0010 air foil sections. The wing tips were shaped as surfaces of revolution by rotating the upper airfoil surface profile through 180 deg about the tip chord line. The delta wings were constructed as plane surfaces with sharp symmetric wedge shaped leading edges having an included angle of 30 deg. All the conventional planforms were untapered with the exception of wing D (aspect ratio 6, half chord line sweep of 19.7 deg). The empirical relationships detailed later exclude the effects of wing taper. However, wing D is still included because experimental data shows[3] that the taper has a relatively small effect on the gust forces as compared to an identical untapered wing.

The oscillatory gust force data are presented in Figs. 1 and 2 as ratios of $\Delta C_L/\bar{w}_g$ and $\Delta C_M/\bar{w}_g$ against a frequency parameter, $\bar{\omega} = \omega c/U$; where ΔC_L and ΔC_M are amplitudes of lift and pitching moment coefficients, $\bar{w}_g$ is the amplitude of gust angle or nondimensionalized "downwash" velocity, ω is the radian gust frequency, c is the wing geometric mean chord, and U is the mean freestream velocity. The results presented here are nondimensionalized with respect to the amplitude of gust oscillation because the original data indicated that for gust incidence variations below stall, the measured force amplitudes behaved linearly with increasing gust amplitude. Thus the results displayed in Figs. 1 and 2 are independent of the wing angle of incidence away from stall. In order to ensure this linearity, the boundary layer on the upper and lower surfaces of the nondelta wings was tripped by sandpaper roughness close to the leading edges. The two delta

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